Subscripts in Expressions

There are times when a subscript does more than just designate a separate variable. For example, if we have the sequence 5, 12, 19, 26, 33, 40,… then we can describe this sequence by saying that the n^{th} item in the sequence is equal to

$$5+(n-1)*7.$$

We could write this as an equation as

$$s_n=5+(n-1)*7.$$

Some note should be made here that such a description is not unique. First, we are using the letter "n" as a placeholder. We could just as easily have used the letter "i", in which case we would have written

$$s_i=5+(i-1)*7.$$

Second, even our formula is not unique; it could just as easily have been written as

$$s_n=-2+7*n, \text{ or as}$$
$$s_n=12+7*(n-2), \text{ or as}$$
$$s_n=-9+(n+1)*7.$$

Each of these would produce the same sequence, namely, 5, 12, 19, 26, 33, 40,…

Third, we have left the specification for the possible values of "n" or "i" as the implied values 1, 2, 3, 4, and so on. If we do not specify the possible values for the subscript then we will assume the values start with 1 and increase from there. On the other hand, we might specify that the values for the subscripts start at 0 and continue increasing. For our sequence 5, 12, 19, 26, 33, 40,…, we might say

s_i=5+7i, where i starts at 0.

Note, however, that if we do this then the first item in the sequence becomes "s sub 0" not "s sub 1".

Let us look at a few examples. For the sequence

s_i=4i+3, i starting at 1

we get the values 7, 11, 15, 19, … In fact, we can say that the sixth item in the sequence, s_6, has the value 4*6+3, or 27, and the fifty-third item in the sequence, s_{53}, has the value 4*53+3, or 212+3, or 215.

For the sequence defined by

s_i=(i+2)/(i+3), i starting at 1,

we get ¾, 4/5, 5/6, 6/7, … For this sequence we find that s_8=10/11.

For the sequence defined by

s_i=4/(2*i-1), where i starts at 1,

we get 4/1, 4/3, 4/5, 4/7, 4/9, … For that sequence we find that s_7=4/(14-1)=4/13 and s_{34}=4/(68-1)=4/67.

We can change that last sequence to make the items alternate signs by modifying the sequence to be

s_i=$(-1)^{(i+1)}$*4/(2*i-1), where i starts at 1

Now we get 4/1, -4/3, 4/5, -4/7, 4/9, -4/11, … In this case s_{20}=$(-1)^{21}$*4/(40-1)=(-1)*4/39=-4/39. This happens to be an interesting sequence because if you found some way to add up all of the items in this sequence the sum would be exactly the value of π. That is, π=4/1 - 4/3 + 4/5 - 4/7 + 4/9 - 4/11 +…

We will try some more examples on the next page.